

Enrollment No: _____

Exam Seat No: _____

C. U. SHAH UNIVERSITY

Winter Examination-2020

Subject Name: Engineering Mathematics-I

Subject Code: 4TE01EMT1

Branch: B.Tech (All)

Semester: 1

Date: 09/03/2021

Time: 03:00 To 06:00

Marks: 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1	Attempt the following questions:	(14)
a)	If the power of x and y both are even then the curve is symmetrical about X-axis (b)Y-axis (c)about both X and Y axes (d)None	01
b)	True/False: the two tangents to the curve $y^2 = x^3$ at the origin are real and distinct.	01
c)	The infinite series $1 + r + r^2 + \dots + r^{n-1}$ is convergent if (a) $ r < 1$ (b) $ r > 1$ (c) $ r \geq 1$ (d) $ r = -1$	01
d)	If $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(x,y)}$ is equal to (a)1 (b)-1 (c)0 (d) None of these	01
e)	Condition for $f(x,y)$ to be maximum (a) $f_x = 0 = f_y, rt < s^2, r < 0$ (b) $f_x = 0 = f_y, rt > s^2, r < 0$ (c) $f_x = 0 = f_y, rt > s^2, r > 0$ (d) $f_x = 0 = f_y, rt = s^2, r > 0$	01
f)	If $u = y^x$, then $\frac{\partial u}{\partial x}$ is (a) xy^{x-1} (b)0 (c) $y^x \log x$ (d)None of these	01
g)	If $f(x,y) = 0$, then the $\frac{dy}{dx}$ is equal to (a) $\frac{f_x}{f_y}$ (b) $\frac{f_y}{f_x}$ (c) $-\frac{f_y}{f_x}$ (d) $-\frac{f_x}{f_y}$	01
h)	The number of solutions to equation $z^2 + \bar{z} = 0$ is (a)1 (b)2 (c)3 (d)4	01
i)	The polar form of the complex number $\frac{1+i}{1-i}$ is (a) $\cos\frac{\pi}{2} + i \sin\frac{\pi}{2}$ (b) $\sin\frac{\pi}{2} + i \cos\frac{\pi}{2}$ (c) $\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}$ (d) $\sin\frac{\pi}{4} + i \cos\frac{\pi}{4}$	01



- j) $\lim_{x \rightarrow 0} \frac{\tan x}{x}$ is of the form 01
 (a) $\frac{0}{0}$ (b) $\frac{\infty}{\infty}$ (c) 0^0 (d) $\infty - \infty$
- k) Which of the following is indeterminate form 01
 (a) 0^0 (b) $0 \cdot \infty$ (c) ∞^∞ (d) All
- l) If $y = x^7$ then $y_7 = \underline{\hspace{2cm}}$. 01
 (a) $7!$ (b) $7!x$ (c) 0 (d) $8!$
- m) If $y = \sin x \cos x$ the y_n equal to 01
 (a) $\frac{1}{2}(2)^n \cos\left(\frac{n\pi}{2} + 2x\right)$ (b) $\frac{1}{2}(2)^n \sin\left(\frac{n\pi}{2} + 2x\right)$
 (c) $\frac{1}{2}(2)^n \sin\left(\frac{n\pi}{2} + x\right)$ (d) None of these
- n) The value of $(i)^i$ is 01
 (a) $e^{-\frac{\pi}{2}}$ (b) $e^{\frac{\pi}{4}}$ (c) e^2 (d) None of these

Attempt any four questions from Q-2 to Q-8

- Q-2 Attempt all questions (14)**
- a. Evaluate: $\lim_{x \rightarrow 1} \left(\frac{x}{x-1} - \frac{1}{\log x} \right)$ 05
- b. If $y = \cos x \cdot \cos 2x \cdot \cos 3x$ then find y_n . 05
- c. Expand $e^{\sin x}$ as a series of ascending power of x up to x^4 . 04
- Q-3 Attempt all questions (14)**
- a. Find roots common to the equation $x^4 + 1 = 0$. 05
- b. Prove that $\cos 6\theta = 32 \cos^6 \theta - 48 \cos^4 \theta + 18 \cos^2 \theta - 1$ 05
- c. Evaluate: $(x, y) \xrightarrow{\lim} (0, 1) \frac{x-xy+3}{x^2y+5xy-y^3}$ 04
- Q-4 Attempt all questions (14)**
- a. If $u = \tan^{-1} \left(\frac{x^2+y^2}{x-y} \right)$ show that $x \left(\frac{\partial u}{\partial x} \right) + y \left(\frac{\partial u}{\partial y} \right) = \frac{1}{2} \sin 2u$. 06
- b. Expand $f(x) = x^4 - 11x^3 + 43x^2 - 60x + 14$ in power of $(x - 3)$. 05
- c. Evaluate: $x \xrightarrow{\lim} 0 \frac{\log x^2}{\cot x^2}$ 03
- Q-5 Attempt all questions (14)**
- a. If $y = a \cos(\log x) + b \sin(\log x)$, prove that
 $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2 + 1)y_n = 0$. 05
- b. Trace the curve $xy^2 = 4a^2(2a - x)$. 05
- c. Test for convergence $\sum_{n=2}^{\infty} \frac{1}{n \log n}$. 04



Q-6	Attempt all questions	(14)
a.	Find the extreme value of the $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.	07
b.	If $u = \sin^{-1}(x - y)$, where $x = 3t$ and $y = 4t^3$, show that $\frac{du}{dt} = \frac{3}{\sqrt{1 - t^2}}$	05
c.	Show that the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{1}{n}\right)$ is convergent.	02
Q-7	Attempt all questions	(14)
a.	Use Taylor's series to expand $\sin x \cos y$ in a power of $\left(x - \frac{\pi}{3}\right)$ and $\left(y - \frac{\pi}{4}\right)$	05
b.	Show that $\sum_{n=1}^{\infty} \frac{n}{2^n}$ converges.	05
c.	Prove that $\cos h^{-1}z = \log(z + \sqrt{z^2 - 1})$.	04
Q-8	Attempt all questions	(14)
a.	If $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$, $z = r \cos \theta$, find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$.	05
b.	Find the maximum value of $V(x, y, z) = xyz$ subject to the constraint $2x + 2y + 2z = 108$	05
c.	Evaluate: $(x, y) \xrightarrow{\lim_{(x,y) \rightarrow (0,0)}} \frac{x^2y}{x^4+y^2}$	04

